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Scaling in self-organized criticality from interface depinning?

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Abstract
The avalanche properties of models that exhibit 'self-organized criticality' (SOC) are still mostly awaiting theoretical explanations. A recent mapping (Alava M and Lauritsen K B 2001 *Europhys. Lett.* **53** 569) of many sand-pile models to interface depinning is presented first, to provide an understanding of how to reach the SOC ensemble and the differences between this ensemble and the usual depinning scenario. In order to derive the SOC avalanche exponents from those of the depinning critical point, a geometric description of the quenched landscape, in which the 'interface' measuring the integrated activity moves, is considered. It turns out that there are two main alternatives concerning the scaling properties of the SOC ensemble. These are outlined in one dimension in the light of scaling arguments and numerical simulations of a sand-pile model which is in the quenched Edwards–Wilkinson universality class.

1. Introduction

The concept of reaching 'self-organized criticality' (SOC) in a model without any apparent tuning parameter is still attracting attention [1]. In many real-life systems, power-law probability distributions are met, and it is thus obvious that the question arises of why they would resemble ordinary critical phenomena. For attempts to answer this question, sand-pile models have provided the prevalent theoretical arena for the last 15 years. Only recently has an understanding finally started to take shape. Two supporting approaches have been developed. The crucial notion is that the critical state in these models draws from both the boundary conditions and the driving force, and also has a generic, field-theoretical description. This can be formulated as a variant of the directed percolation-style Reggeon field theory (RFT) [2], or as a mapping to *interface depinning* [3–6]. The gist of this mapping is based on the description of the *history* of the sand-pile model and its dynamics via a stochastic differential equation, the quenched Edwards–Wilkinson (qEW) equation [7, 8]. Likewise a suitable RFT for sand-piles includes of necessity a density-conserving term that accounts for the effects of diffusional transport of particles or grains.

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In this paper the description of sand-piles through interface depinning is reviewed. The issue of particular interest to us is the physics of driven interfaces that describe sand-pile models. With this in mind, in section 2 we briefly introduce the mapping, and discuss the ensemble in which SOC is reached. It is seen that this is not any of the normal ones familiar in the context of the qEW equation, say. The next section is devoted to a discussion of the standard properties of interface scaling at the depinning transition, and, correspondingly, the scaling laws usually formulated for sand-piles depending on the ensemble. We next concentrate in particular on a *geometric description* of depinning. This is the essential issue in obtaining the scaling exponents of SOC avalanches: the extension of the theory of 'elastic pinning paths' to the SOC ensemble. In section 4 numerical results are considered for the one-dimensional qEW sand-pile in the SOC case. This is the *simplest* model system in which one can try to extract a correspondence with the avalanche and the depinning pictures. This is since the 1D pinning paths can be discussed without the introduction of extra, independent exponents. Finally, section 5 finishes the paper with a summary of SOC properties based on recent advances including the numerical work presented here, and of remaining open problems.

2. Mapping sand-piles to interfaces

Consider a sand-pile model with each site x of a hypercubic lattice having $z(x, t)$ grains. When $z(x, t)$ exceeds a critical threshold $z_c(x)$, the site is active and topples, so (y is a nearest neighbour of x)

$$z(x, t + 1) = z(x, t) - 2d \qquad z(y, t + 1) = z(y, t) + 1. \tag{1}$$

$z_c(x)$ is taken to be a random variable, chosen from a probability distribution $P(z_c)$ again *after each toppling*. If there are no active sites in the system, one grain is added to a randomly chosen site, $z(x, t) \to z(x, t) + 1$. This is the SOC ensemble, defined via the driving force and the open boundaries, characterized by avalanches that may ensue after a grain has been deposited. The dynamics of this model can be reinterpreted through an 'interface' or history $H(x, t)$, which follows the memory of all the activity at x. H counts topplings at site x up to time t, and has the dynamics

$$H(x, t + 1) = \begin{cases} H(x, t) + 1 & f(x, t) > 0 \\ H(x, t) & f(x, t) \leqslant 0. \end{cases} \tag{2}$$

This can be written as a discrete interface equation:

$$\frac{\Delta H}{\Delta t} = \theta(f(x, t)) \tag{3}$$

with $\theta(f)$ the step function forcing it such that the interface does not move backwards. The 'local force' is

$$f(x, t) = z(x, t) - z_c(x) = n_x^{\text{in}} - n_x^{\text{out}} - z_c(x) \tag{4}$$

in terms of n_x^{in} (grains added to site x up to time t) and n_x^{out} (grains removed from x). The fluxes n_x^{in} and n_x^{out} can be worked out in terms of the local height field $H(x, t)$, and a columnar force term $F(x, t)$ which counts the number of grains added to site x by the external driving force:

$$f(x, t) = \nabla^2 H + F(x, t) - z_c(x, H). \tag{5}$$

The step function, $\theta(f)$, in equation (3), the condition that the interface does not move backwards, introduces an extra noise term $\sigma(x, H)$—the velocity of the interface is either one or zero—but for the current example this should not be relevant. Combining the three sources of effective noise, F, z_c, and σ, one ends up with the discretized interface equation

$$\frac{\Delta H}{\Delta t} = \nabla^2 H + \eta(x, H) + F(x, t) + \sigma(x, H). \tag{6}$$

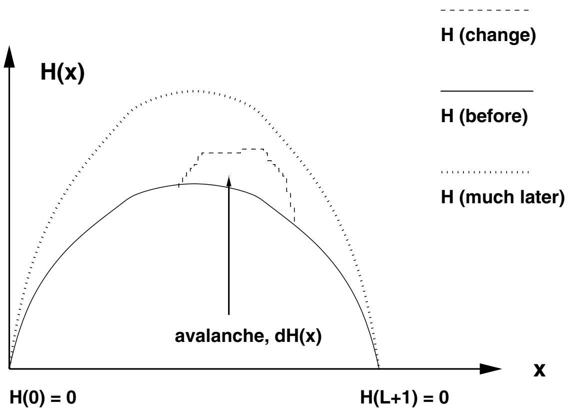

Figure 1. A one-dimensional schematic example of the interface or history representation of a SOC sand-pile model. The mean-field interface follows an average parabolic shape, which also implies that in the SOC steady-state $v(x)$ is parabolic. Notice the boundary conditions $H = 0$ that ensures the loss of particles (equalling the increased elastic energy) in that state.

Here the quenched noise $\eta(x, H) = -z_c(x, H)$, and we have obtained the central difference discretization of a continuum diffusion equation with quenched noise, called the linear interface model (LIM) or the quenched qEW equation [7, 8]. The continuum limit reads $\partial_t h(x, t) = \nu \nabla^2 h(x, t) + F + \eta(x, h(x, t))$.

The SOC ensemble is illustrated in figure 1. Once the local force is increased, by adding a grain and making $F(x, t) \to F(x, t) + 1$ at the chosen x, an avalanche starts since the force overcomes the pinning force, $\nabla^2 H + \eta(x, H) + F(x, t) > 0$. The interface at x moves one step, $\Delta H(x) = 1$. In the subsequent dynamics of the avalanche the columnar force term $F(x)$ does not change. For the SOC sand-piles, the correct choice of the interface boundary condition is $H = 0$ which is to be imposed at two 'extra sites' ($x = 0, L + 1$ for a system of size L in 1D). The ever-increasing $\langle F(t) \rangle$ leads to an average parabolic interface shape via the cancellation of the Laplacian by F. It is to be stressed that this implicitly contains the physics of the SOC state: the driving force $F(x, t)$ is increased so slowly that the avalanches do not overlap and are therefore well defined [3].

3. Scaling properties of ensembles

Equation (6) exhibits a *depinning transition* at a threshold force F_c in a 'normal' ensemble. The interface configuration and dynamics develop critical correlations in the vicinity of the critical point. For the case of point-correlated disorder the normal way to analyse the LIM is to use the functional renormalization group method. One-loop expressions for the exponents are found in papers by Nattermann *et al* [7], Narayan and Fisher [8], and in the more recent ones by Le Doussal and collaborators [9].

The problem is technically and from the fundamental physics viewpoint difficult, since the whole disorder correlator is renormalized. The point-random-field noise term forms one of the universality classes in this problem [7, 8]; others are the LIM with columnar noise [10] and the quenched Kardar–Parisi–Zhang (KPZ) equation [11–14] in more general terms. The mapping of SOC models to variants of the LIM holds some surprises, however. For instance, while the conjecture that the Manna model [15] should be in the usual point-disorder LIM class seems to be borne out in two dimensions, this is clearly not so in one dimension if one is considering the depinning ensemble [16]. The complications arise since an arbitrary choice of sand-pile rules can lead to *non-standard noise correlations* that do not need to have *a priori* the same RG fixed point as the random-field or random-force case.

In interface language, the relevant exponents for describing the depinning phase transition are ν (the correlation length exponent), z (the dynamical exponent), and χ (the roughness exponent). Moreover, assuming that the avalanche dynamics suffices to describe the interface dynamics off the critical point, $\theta = \nu(z - \chi)$ holds for the velocity exponent, with $v \sim (F - F_c)^\theta$ [7, 8]. For point-like disorder, the first-loop functional RG results cited above read $\chi = (4-d)/3$ and $z = 2-(4-d)/9$. Notice the exponent relation $\nu = 1/(2-\chi)$ which, together with the θ-exponent relation, manifests the fact that there is only one temporal and is only one spatial scale at the critical point.

The typical quantity to be measured in the interface context is the interface width w (mean fluctuation) which in most cases equals other measures like the two-point correlation function and the structure factor [13]. From the sand-pile viewpoint, these measure the correlations and fluctuations in the activity history—that is, in the avalanche series. Initially the width grows, as a function of time, as $w \sim t^\beta$ defining the 'growth exponent' β until either saturation is reached (with a finite order parameter, v), or the interface gets pinned at or below F_c. Scaling now implies $\beta z = \chi$, as for general interface models.

The LIM exhibits an important invariance, with the static response scaling as [8] $\chi(q, \omega = 0) \sim q^{-2}$, so

$$\gamma/\nu = 2. \tag{7}$$

For forces below F_c, the (bulk) response of the interface triggered by a small increase in F is $\chi_{\text{bulk}} \equiv \mathrm{d}\langle H\rangle/\mathrm{d}F \sim (F_c - F)^{-\gamma}$. On assuming that the avalanche due to a point seed scales as $\Delta H \sim s \sim \ell^D$, $D = d + \chi$ (since H scales with the roughness as ℓ^χ), a hyperscaling relation can be derived for γ. Right at the critical point [7, 8], the roughness of the interface scales as ℓ^χ and, assuming that $\Delta\langle H\rangle$ will scale in the same way, it follows that

$$\gamma = 1 + \chi\nu. \tag{8}$$

This also implies $\chi + 1/\nu = 2$, as noted above. The standard scaling relations are valid for parallel dynamics: all sites with $\partial H/\partial t > 0$ are updated in parallel. For extremal driving force criticality (updating one unstable site at a time), the dynamic exponent reads $z_{\text{ED}} = 2$.

The 1D LIM is a bit more peculiar than one might expect. Numerical findings (which have recently been matched by the two-loop RG results of Le Doussal and co-workers) imply that $\chi \sim 1.2$–1.25 which is larger than the one-loop and scaling argument result $\chi_{1d} = 1$. The physical interpretation of the fact that $\chi > 1$ has been dubbed 'anomalous scaling' [17, 18], and arises from a divergent mean height difference between neighbouring sites, with $t \to \infty$.

The SOC steady state is characterized by the probability of having avalanches of lifetime t and size s which follow power-law distributions: $p(t) = t^{-\tau_t} f_t(t/L^z)$ and $p(s) = s^{-\tau} f(s/L^D)$, with $s \sim t^{D/z}$ and $z(\tau_t - 1) = D(\tau - 1)$. One can also characterize the avalanche by its linear dimension, $p(\ell) = \ell^{-\tau_\ell} f_\ell(\ell/L)$, with $\tau_\ell = 1 + D(\tau - 1)$. Here the size scales as $s \sim \ell^D$ and the (spatial) area as ℓ^d (for compact avalanches) with ℓ the linear dimension. The fact that each added grain will perform of the order of L^2 [19] topplings before leaving the system leads to the fundamental result

$$\langle s\rangle \sim L^2 \tag{9}$$

independent of dimension [20]. Thus, $\tau = 2 - 2/D$ and $\tau_t = 1 + (D - 2)/z$. Equation (9) yields $\gamma/\nu = 2$, where γ describes the divergence of the susceptibility (bulk response to a bulk field) near a critical point, if one generalizes the exponent relation of the depinning ensemble to the SOC case. In the particular case of an increasing driving force and a bulk dissipation, which induces a term $-kH$ in the qEW equation, this should, evidently, be valid [21].

4. The one-dimensional QEW sand-pile: geometric description of avalanches

To go beyond the scaling exponents to a description of the *probability distributions* at the critical point of any particular ensemble is a more challenging task. This is easiest in one dimension, which we thus discuss here. The most well-known case, in which the geometry of the random quenched landscape allows one to use a self-affine picture of the progress of an interface through it, is given by directed percolation depinning (DPD), the quenched KPZ cousin of the qEW equation [11–13].

In this language the interface moves, e.g. in the case of an applied extremal driving force, via a succession of 'punctuation events'. The interface is assumed to invade the voids of a connected network in each of these events, in 'bursts'. In between these events, the pinned interface is mapped into a connected path on the backbone of a suitable (elastic) percolation problem [22]. For DPD the analogy is more or less clear, and for the qEW equation one speaks of *elastic pinning paths*. Such paths have the characteristics that the RHS of the qEW equation is always negative semi-definite, i.e. $f \leqslant 0$. It is not known rigorously whether such paths at criticality follow strictly the DPD-like scaling properties. There are two fundamental issues: the geometric properties of avalanches (the scaling of voids, or whether the relation of size to area can be characterized with a 'local' roughness exponent $\chi_{\rm loc}$) and the probability of inducing an avalanche, or 'punctuation event', if the interface is pushed at any particular spot (this issue is still open for discussion; see [23, 24, 26]).

Assuming that the DPD analogy works [23], it follows in the depinning ensemble for the avalanche size exponent that

$$\tau_{s,\rm dep} = 1 + (1/(1 + \chi_{\rm loc}))(1 - 1/\nu) \tag{10}$$

which using e.g. the exponent relation $\nu = 1/(2 - \chi)$ and a reasonable value for $\chi_{\rm loc}$, taken to equal the global χ, produces

$$\tau_{s,\rm dep} \approx 1.08. \tag{11}$$

For the SOC ensemble, the description of the critical state is given in terms of the avalanche distribution exponents—note that due to the inhomogeneity of the ensemble, e.g. pertubing the system off the critical point is more complicated [5]. One has then to ask the question of what the prediction of the pinning-path picture is. Due to the symmetries (static response) of the qEW equation, it could be assumed that the typically parabolic interface profile is irrelevant, except perhaps for finite-size corrections induced by the boundary condition that is imposed on the pinning paths since $H = 0$ at the edges. This would imply, in particular, that for the SOC avalanche exponent it is the case that

$$\tau_{s,\rm SOC} \rightarrow \tau_{s,\rm dep}. \tag{12}$$

A parallel approach is to use the invariant (9) which leads to the prediction

$$\tau_{s,\rm SOC} = 2 - 2/D = 2 - 2/(1 + \chi_{\rm loc}) \tag{13}$$

on using the roughness exponent to estimate the cut-off dimension of avalanches. This is identical to the pinning-path estimate above.

These are then the major issues: is the depinning geometric description applicable to the SOC ensemble and if not, why not? The answer should depend only on fundamental similarities or differences between the ensembles, and not on the particular model—the class of qEW-like models—or the dimensionality. The study of the outcome is easiest in 1D, whereas in higher dimensions further independent exponents are needed, i.e. assumptions to describe the probability distributions, since the avalanches have, in addition to an area versus volume relation, also a perimeter length versus area one [24, 25].

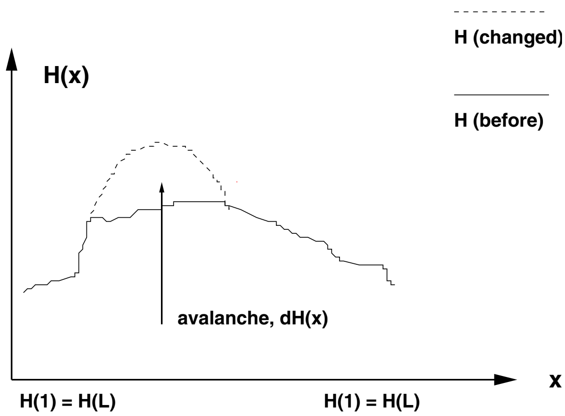

Figure 2. An avalanche at the critical point of the normal, translationally invariant, 'depinning ensemble'. The statistical description of such avalanches follows the scaling law $\langle s\rangle = l^{1+\chi}$, where l is (here in 1D) the area of the avalanche and s its size.

To study this issue, we next outline some numerical results on the 1D qEW equation/LIM, obtained from a Leschhorn-like cellular automaton [17, 26] for the interface problem. The system is run as a sand-pile in that after the interface gets pinned a new avalanche initiation is effected by increasing the local force F_i at a random location by one. System sizes up to $L = 8192$ have been studied, with 2×10^6 avalanches for the largest sizes. The resulting avalanche size distributions, after logarithmic binning, imply that the effective τ-exponent varies with L, and that the weight of the power-law-like tail increases with the size. Meanwhile, the effective roughness exponent slightly *decreases*. Due to the systematic finite-size corrections, it does not make sense to try a normal data collapse using a fixed D and τ_{SOC} for all L. This is a little bit surprising given that relation (9) is fulfilled within numerical accuracy by the data, and that higher momenta of the size distributions indicate just simple scaling of the avalanche size distribution. Certainly, more numerical analysis is called for, but two facts are worth pointing out. *First*, even for the largest system size the effective τ is way off from the 'predicted' 1.08 (1.02 for $L = 8192$). *Second*, by blindly—without any *a priori* justification—using the scaling ansatz

$$\tau_{SOC}(L) = \tau_{SOC}(L = \infty) + \Delta\tau(L) \tag{14}$$

with a 'best-working' function $\Delta\tau$, one observes that there is an apparent power-law correction to the exponent, which extrapolate very slowly in L to $\tau_{SOC}(\infty) \sim 1.04$, i.e. clearly off the depinning ensemble value. Note that τ_{dep} should on the other hand give an upper limit for the SOC exponent. From τ_{SOC} it would be in principle possible to derive the other exponents as well, using the consequences of equation (9),

5. Conclusions

Above, we have discussed a strategy for understanding so-called 'self-organized criticality' by mapping the history of a sand-pile model to a driven interface in a random medium. For SOC models this idea is useful since it helps us to understand universality classes (via noise terms generated), though there are theoretical challenges in the understanding of the possible classes (RG fixed points), and in the role of the 'discretization' (called σ-noise above). Such work follows the historical connections of SOC to depinning, and extends it by explicitly

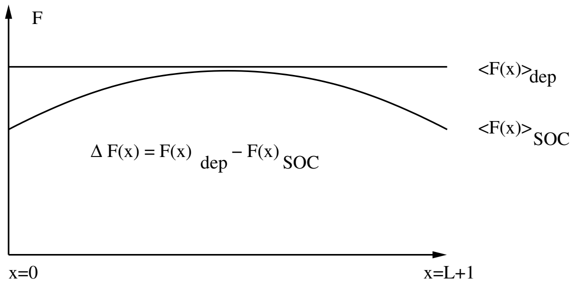

Figure 3. The *average* force per site, $\langle f(x)\rangle$, in the depinning and SOC ensembles. $\langle f\rangle$ is to be computed by computing the number of grains per site between the avalanches.

constructing the right ensemble for reaching SOC, and by outlining a general strategy for understanding various models. Applied to other absorbing state phase transitions, in general, the history/interface description should be of interest. In some cases (e.g. the contact process) it can give rise to new, seemingly independent exponents [27].

Once one has defined the right ensemble for SOC in interface depinning, the most pertinent question becomes that of whether the usual avalanche exponents in SOC can be derived from those of the depinning transition. Here we have addressed the question, but since the outcome is still open, it is worth reiterating the two possibilities. Either, eventually, on studying large enough systems numerically, the exponents of the statistically homogeneous ensemble are recovered once finite-size effects become negligible; or it becomes apparent that the SOC ensemble *is an independent one*. The microscopic reason for this is that the density of grains (average force for the interface) is non-uniform: in one dimension it is easy to see that a site x will get a larger grain flux from its neighbour on the bulk side and a smaller one from the boundary side—more trivially, there is a net flux of grains towards the boundaries. This inhomogeneity may persist in the thermodynamic limit, in which case the avalanche relations will be determined by its scaling properties.

Consider the idea depicted in figure 3. The integral of the average force deviation x_i, $\Delta F(x) = F_{\rm dep}(x) - F_{\rm SOC}(x)$, where $F_{\rm dep}$ and $F_{\rm SOC}$ are averages at the critical points of the ensembles, can be used to define a finite-size correction to the normal critical point, as $\int \Delta F(x)\,{\rm d}x \equiv \delta F(L)$. It is seen that δF will be dependent on the exact scaling function of $F(x)_{\rm SOC}$, whose computation is thus the crucial issue. It will give indirectly the *true* correlation length exponent $\nu_{\rm SOC}$ in the SOC ensemble via the L-dependence of the finite-size correction, which may or may not be the same as for the depinning case. The implication can be recast such that the usual exponents like ν are derived from the RG in an ensemble with statistical translational invariance in the x-direction. It is thus not obvious whether the properties of a self-organized critical state actually follow from boundary-induced criticality or are related to the usual depinning scenario. In this respect the usual SOC models discussed here are inherently more complicated than the boundary-driven cases like the so-called 'rice-pile' model. This is in particular true if the symmetries of the depinning transition are broken by the SOC ensemble, as is the case for the quenched KPZ equation [14].

To summarize, the above problem is central in understanding SOC-like systems. It may also be tackled, perhaps, from the viewpoint of absorbing state phase transitions and their field-theoretical description which provides a parallel route to the interface one. There are many other interesting issues, such as the early-stage dynamics (growth exponent β, suitably defined for the SOC ensemble), the question of the pinning path/manifolds in higher dimensions, and so forth. For all of these it is invaluable that one can use continuum descriptions of the SOC sand-piles that also seem to be inter-connected in an intriguing fashion [28, 29].

Acknowledgments

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